Evaluation of the Superconductor Limit-Line Criterion in Corsica

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Introduction

The "limit-line" criterion defines the allowable operating region of a superconducting coil to be below a line in B–I space, where B is the peak instantaneous field in a coil and I is the corresponding conductor current. We implement this criterion in Corsica by defining a *utilization factor*, u, which is a measure of the operating point distance from the origin normalized to the limit-line distance from the origin. Thus, $u < 1$ indicates a coil is within its allowed operating space, $u = 1$ indicates a coil is at its operational limit, and $u > 1$ means a coil is above its allowed range.

In the following sections we first derive an expression for the utilization factor, then describe how to use Corsica to accurately evaluate utilization factors and other coil diagnostic quantities.

Utilization Factor

The utilization factor is a measure of the distance of the operating point in B–I space from the origin, relative to the limit-line distance from the origin. Representing the normal distance from the operating point to the limit-line with d and the normal distance from the origin to the limit-line with d_0, our definition of the utilization factor is

$$u = 1 - \frac{d}{d_0}. \tag{1}$$

In the next subsection we derive expressions for the distances d and d_0.

Normal Distances

We will work in general Cartesian coordinates where our limit-line will be defined by two points, P_1 and P_2, with the operating point labelled P.

We begin by defining a vector $\mathbf{v}$ to represent the limit-line segment from P_1 to P_2,

$$\mathbf{v} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \end{vmatrix}, \tag{2}$$

and its normal vector,

$$\mathbf{v}_\perp = \begin{vmatrix} y_2 - y_1 \\ -(x_2 - x_1) \end{vmatrix}. \tag{3}$$

The unit normal to $\mathbf{v}$ is then

$$\hat{\mathbf{n}} = \frac{\mathbf{v}_\perp}{L}, \tag{4}$$

where L is the magnitude $||\mathbf{v}||$ or $||\mathbf{v}_\perp||$.

Using $\mathbf{r}$ to represent the vector from our operating point, P, to one end of the limit-line, say P_1:

$$\mathbf{r} = \begin{vmatrix} x_1 - x \\ y_1 - y \end{vmatrix}, \tag{5}$$

we can write the *signed* normal distance from the operating point to the limit-line by projecting $\mathbf{r}$ onto $\hat{\mathbf{n}}$, yielding

$$d = \mathbf{r} \cdot \hat{\mathbf{n}} = \frac{(x_1 - x)(y_2 - y_1) - (y_1 - y)(x_2 - x_1)}{L}. \tag{6}$$

Similarly, the vector from the origin to P_1 is

$$\mathbf{r_0} = \begin{vmatrix} x_1 - 0 \\ y_1 - 0 \end{vmatrix}, \tag{7}$$

and the distance from the origin to the limit-line is therefore

$$d_0 = \mathbf{r_0} \cdot \hat{\mathbf{n}} = \frac{x_1(y_2 - y_1) - y_1(x_2 - x_1)}{L}. \tag{8}$$

In the next subsection we combine these distances and express the result in terms of coil variables.

Expression for the Utilization Factor

We can now write the utilization factor by substituting Equations 6 and 8 into Equation 1, resulting in

$$u = 1 - \frac{d}{d_0} = \frac{x(y_2 - y_1) - y(x_2 - x_1)}{x_1(y_2 - y_1) - y_1(x_2 - x_1)}. \tag{9}$$

Since the limit-line is defined by two points, we choose to use the axis intercepts for P_1 and P_2: $(0, Y)$ and $(X, 0)$, which reduces Equation 9 to

$$u = \frac{xY + yX}{XY}. \tag{10}$$

In terms of coil variables, with $x \to B$ and $y \to I$, our final equation for the utilization factor is

$$u = \frac{B_{max} I_{lim} + |I_{cond}| B_{lim}}{B_{lim} I_{lim}}, \tag{11}$$

where B_{max} is the peak field, I_{cond} is the conductor current, and with B_{lim} and I_{lim} representing the limit-line axis intercepts.

In the next section we describe how to accurately evaluate utilization factors and other coil diagnostic quantities in Corsica.

Utilization Factor Evaluation in Corsica

In order to evaluate the utilization factor (Equation 11) in Corsica we need a good estimate of the peak B-field, B_{max}, in each coil in addition to its operating current, which is readily available from an equilibrium solution. In Corsica the peak fields in the coils are diagnostic quantities evaluated when: (a) the Corsica coil diagnostics switch `lop0` has been set to one, and (b) the coil gridding resolution parameter array, `ngp`, has been set with appropriate values. These input parameters and other relevant[1] coil input quantities are listed in Table 1. Note that total coil current, NI,

Table 1: Corsica coil input quantities

Quantity	Corsica *Name*	*Units*	*Description*
switch	`lop0`	—	Coil diagnostics switch (0: off, 1: on)
N_c	`nc`	—	Total number of coils
N_{PFC}	`npfc`	—	Number of driven coils
N	`ntc(nc)`	—	Number of turns
NI	`cc(nc)`	MA	Total coil current
R_c	`rc(nc)`	m	Mean coil major radius
Z_c	`zc(nc)`	m	Mean axial position
ΔR_c	`drc(nc)`	m	Radial build
ΔZ_c	`dzc(nc)`	m	Axial build
α_c	`ac(nc)`	rad	Type-1 parallelogram angle
$\alpha_{2,c}$	`ac2(nc)`	rad	Type-2 parallelogram angle
$n_{\Delta R_c}$	`nrc(nc)`	—	Number of radial filaments (across ΔR_c)
$n_{\Delta Z_c}$	`nzc(nc)`	—	Number of axial filaments (across ΔZ_c)
n_{Bgrid}	`ngp(nc)`	—	Coil gridding parameter
B_{lim}	`blimc(nc)`	T	Limit-line B-axis intercept
I_{lim}	`ilimc(nc)`	A	Limit-line I-axis intercept

although generally an output from an equilibrium calculation, may optionally be specified by the user so it is included in Table 1. (The conductor current, I, is a derived quantity—it is used internally in evaluating coil diagnostics.) Table 2 lists some of the Corsica coil output quantities.

[1] Tables 1 and 2 list subsets of coil input and output quantities; use the Corsica `list` command or the `attrlist` routine for more information; for example, enter `list TeqGS_input` or `attrlist(stdout, "input")` in a Corsica session.

Table 2: Corsica coil output quantities

Quantity	Corsica *Name*	*Units*	*Description*
NI	`cc(nc)`	MA	Total coil current
$\langle B_R \rangle$	`pfbr(nc)`	T	Mean radial field
$\langle B_Z \rangle$	`pfbz(nc)`	T	Mean axial field
B_{max}	`pfbc(nc)`	T	Peak field
F_r	`pffr(nc)`	MN	Total radial force
F_z	`pffz(nc)`	MN	Total axial force
M	`pfim(nc,nc)`	μH	Coil inductance matrix
u	`ufc(nc)`	—	Utilization factor

Coils are "discretized" for two purposes in Corsica, for: (1) plasma equilibrium analysis and in the circuit equations (i.e., the coil-plasma and coil-coil Green's functions, evaluated with a *filament model* for the current source) and (2) optionally for coil diagnostics (i.e., the coil-coil Green's functions, evaluated by integrating the Biot-Savart equation over regions of uniform current density; sometimes called the *solid coil model*). The Biot-Savart coil-coil Green's functions are used to evaluate the field and force distribution throughout each coil, in addition to the peak field, a vital part of utilization factor determination. The following two subsections describe the two approaches to modeling the coil-current distribution in Corsica.

Coil Model for Equilibrium Analyses

The coil filament distribution for equilibrium analyses is a uniform 2D rectangular (or parallelogram[2], if angle α_c or $\alpha_{2,c}$ is non-zero) arrangement of filaments, specified with arrays `nrc` and `nzc`, the number of filaments across the radial and axial coil winding pack dimensions. If `nrc` or `nzc` are zero, default values will be provided by Corsica. However, it is recommended they be specified by the user after a systematic sensitivity evaluation for each new machine configuration to provide the desired accuracy of the plasma equilibrium solution.

Gridding for Coil Diagnostics

The subdivision of the winding pack (the bundle of turns) for coil diagnostics is determined by the input array `ngp`, which specifies the number of field-evaluation grid points, $n_{Bgrid}(i)$, across the smallest dimension of the ith coil, with resolution:

$$\Delta s(i) = \frac{min(\Delta R_c(i), \Delta Z_c(i))}{n_{Bgrid}(i) - 1}.$$

Since peak B-fields usually occur on the surface of the winding pack, we require that `ngp` be 2 or more if coil diagnostics are desired as we always want field evaluation points on the surfaces of the winding pack. Diagnostics will not be evaluated

[2] Coils with parallelogram cross-sections are described as Type-1 ($\alpha_c \neq 0$) or Type-2 ($\alpha_{2,c} \neq 0$) and rectangular coils have $\alpha_c = \alpha_{2,c} = 0$, following the convention used at General Atomics (see `http://fusion.gat.com/theory/Efit`).

with Biot-Savart integration if ngp is zero for a coil[3]. The number of grid points in the longer cross-sectional dimension is the rounded value of:

$$n_{Bgrid,long}(i) = \frac{max(\Delta R_c(i), \Delta Z_c(i))}{\Delta s(i)} + 1.$$

The user must determine the value of ngp for each coil which provides the desired accuracy of the results, setting ngp to zero for those coils where diagnostic output is not desired (e.g., non-superconducting coils, coils representing bus-work, etc.).

In contrast to the filament model for coil-plasma Green's functions, the Corsica Biot-Savart algorithm demands coil cross-sections be rectangular. Therefore, ngp will be coerced to zero for any coils where the Type-1 or Type-2 parallelogram angles are non-zero.

In earlier versions of Corsica, ngp was a scalar parameter applied to all coils and the coil field computational parameter, dfac[4], had the default value of 10 (it is now 10^6 by default). To achieve the same coil gridding arrangement with the present code, one can reproduce this old behavior by entering the following to Corsica:

```
# Reproduce 'old' code behavior
integer ngp_old = 5 # The old default value was 5
real ds = min(min(drc), min(dzc))/ngp_old
ngp = min(drc, dzc)/ds + 0.5
dfac = 10
```

Use of the routine described below is recommended to generate values of ngp as it provides a more rigorous determination of ngp for each coil based on a user-specified accuracy criterion.

Systematic Coil Gridding

A built-in script routine is available in Corsica to set the values of ngp for each coil based on an accuracy criterion for the peak field. In order to use it, first initialize all values of ngp to two, except for those coils where coil diagnostics are not desired (where ngp must be left at its default value of zero). Then, execute the routine with an optional argument specifying the desired *absolute* field accuracy, in units of Tesla, for example:

[3] Actually, coil magnetic fields will still be evaluated if $n_{Bgrid} = 0$ for all driven (n_{PFC}) coils, when the switch lop0 is set to one, using the $n_{\Delta R_c} \times n_{\Delta Z_c}$ filament model. The value $n_{Bgrid} = 1$ is reserved for signaling a special grid layout, also using the filament model, for use in modeling coils with ferromagnetic elements.

[4] The "distance factor", dfac, is used to trigger a switch within the solid-coil Biot-Savart model from a finite current-density model to a single-filament model when the current source is more than dfac units of coil cross-sectional size from the field evaluation point, to reduce computational time. This switching between models is no longer necessary with present-day computer speeds and it is therefore recommended that dfac not be changed from its present default value of 10^6, which essentially eliminates any use of the single-filament approximation in evaluating B-fields in coils.

```
ngp = 2; ngp(npfc+1:nc) = 0
call set_ngp(1e-3)
```

The default value of the field accuracy parameter is 0.01 T, which, for peak field criteria of 10-15 T, implies a relative error criterion of about 10^{-3}. This routine simply increments each value of ngp until the present value of B_{max} for a coil differs from its previous value by less than the specified accuracy.

Plot Utilization Factors

The Corsica built-in graphics routine, pufc, is available to display utilization factors and list pertinent coil diagnostic quantities. It plots the $(I/I_{lim}, B_{max}/B_{lim})$ points for the coils relative to the Limit-Line.

Summary

We have described the superconductor *B–I* limit-line criterion and conductor utilization factor evaluation (Equation 11) as implemented in Corsica. The conductor utilization factor measures the relative distance of the operating point to the limit-line, thus: $u = 1$ represents a coil at its limit. The utilization factor and other coil diagnostics are evaluated after following the steps below:

1. Turn on coil diagnostics by setting switch lop0 to one. The diagnostic quantities are evaluated after an equilibrium calculation.
2. Specify the coil gridding parameter array ngp values at two or more for coils where *B*-fields are desired. The built-in routine set_ngp can be used to determine values of ngp to achieve a desired accuracy. This step will trigger the evaluation of coil fields and forces using the Biot-Savart equation.
3. To evaluate the utilization factor (array ufc), specify the number of turns, ntc, and the limit-line axis intercepts in blimc and ilimc.
4. Optionally, use ufc as a constraint in the ceq package.
5. Display utilization factors with the built-in pufc routine.

Coil diagnostics are available after a free-boundary equilibrium calculation. The utilization factor can be used by the Corsica constrained equilibrium solver (package ceq) to run selected coils up to their limit (e.g., for initial magnetization and end-of-burn states or any other equilibrium states where one wants to constrain coils at their limits), by setting the constraint name to a ufc element and the corresponding constraint value to one.