Simulation of magneto-hydrodynamic (MHD) flows: electric potential formulation

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Outline

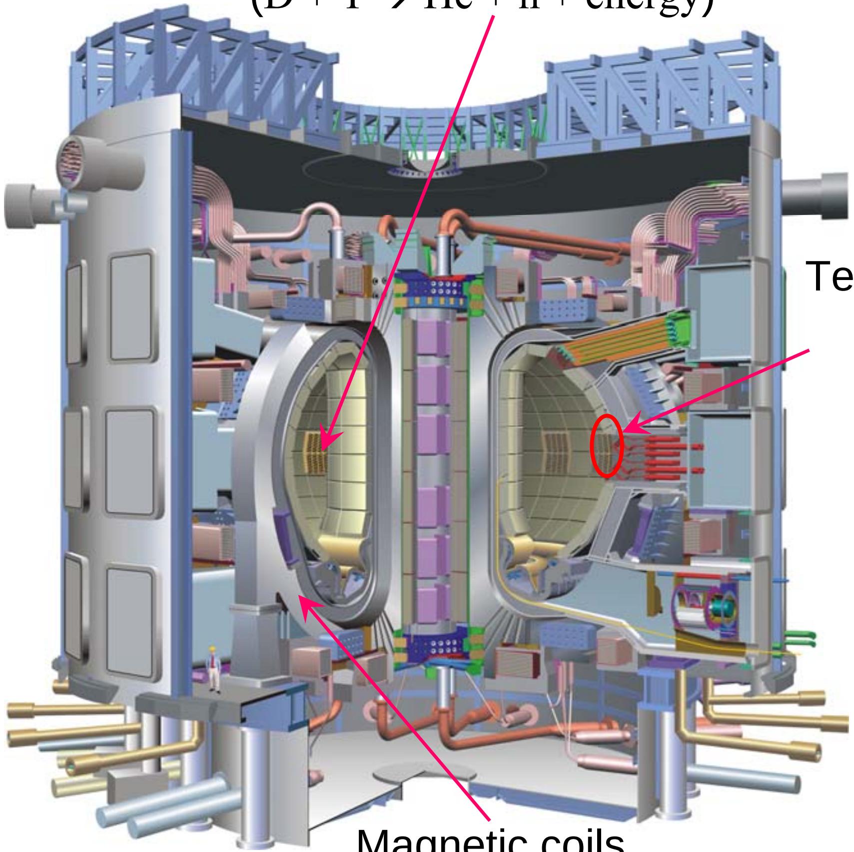
- Motivations for studying MHD flows
- Why a formulation with electric potential?

  available *mhdFoam* → <u>new solver</u> *mhdEpotFoam*
- Magneto-hydrodynamic (MHD) equations
- Issues for MHD flow simulations

  → Mesh constraints
  → Numerical algorithm
- MHD flow in electrically conducting ducts
  → <u>new solver</u> *conjugatemhdFoam*
- MHD flows in ducts: solver validation
- Summary & Outlook

Liquid metal flows in fusion blankets

Fusion plasma
$(D + T \rightarrow He + n + \text{energy})$



Test blanket module (TBM)

Magnetic coils

Fusion blanket

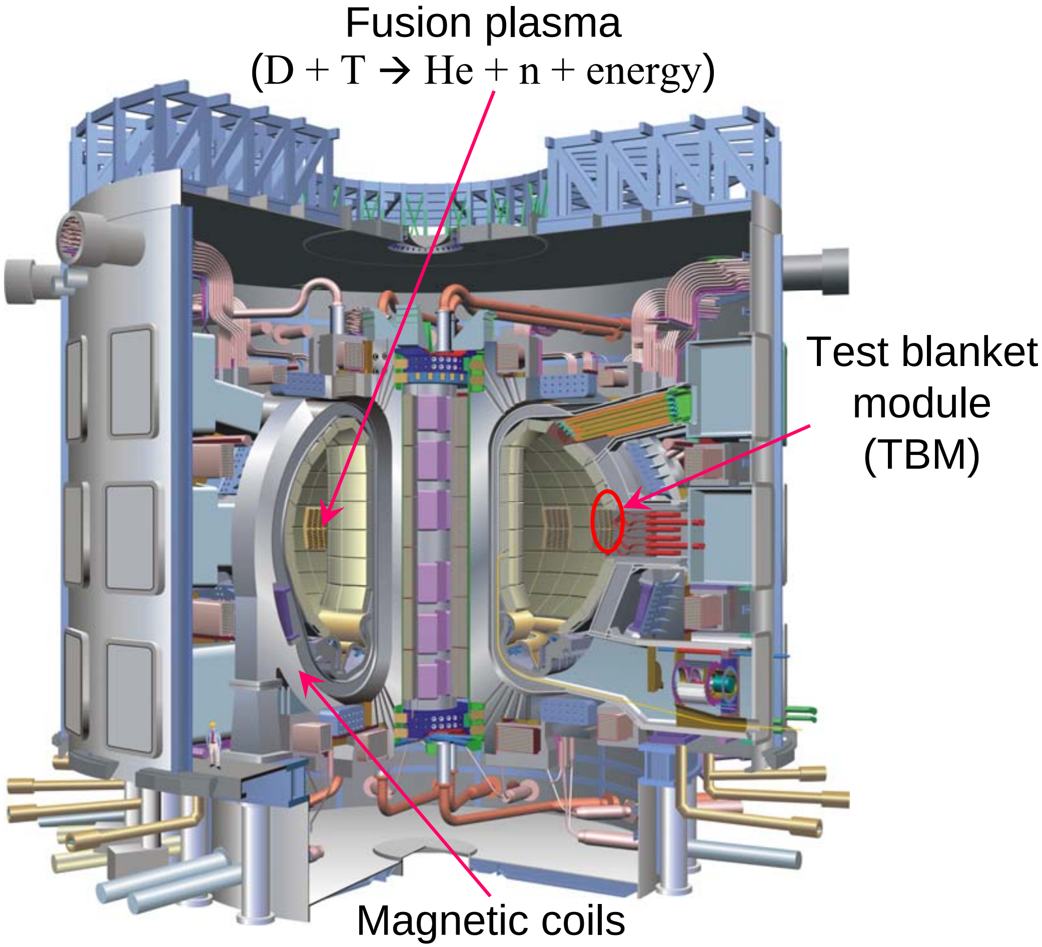
- Radiation shielding
- Breeding of tritium
  $^{6}Li + n \rightarrow He + T + \text{energy}$
- Heat removal: conversion of nuclear kinetic energy into electric energy

Requirements can be accomplished with Li-containing liquids as breeder and coolant

***I**nternational **T**hermonuclear **E**xperimental **R**eactor*

⇒ ***Magnetic confinement of plasma***

Liquid metal flows in fusion blankets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Moving electrically conducting fluid ←→ magnetic field

Liquid metal magneto-hydrodynamics (MHD)

Liquid metal flows in fusion blankets

Other MHD applications:

- ❖ Development of measuring techniques in liquid metal flows, electromagnetic flow meters and pumps.
- ❖ Metallurgical technology – continuous casting (surface treatment, MHD liquid metal stirring), industrial processes…

Focus on fusion applications:

- ➢ **High magnetic fields** ($\mathbf{B} = 4 \div 11$T)
  → very thin MHD boundary layers
- ➢ **Coupled phenomena**
- ➢ **Complex geometries**

⇒ **Strict numerical issues and requirements**

Simulation is a challenging task!

Fusion blanket

- ➢ Radiation shielding
- ➢ Breeding of tritium
  ${}^{6}\mathrm{Li} + \mathrm{n} \rightarrow \mathrm{He} + \mathrm{T} + \mathrm{energy}$
- ➢ Heat removal: conversion of nuclear kinetic energy into electric energy

Requirements can be accomplished with Li-containing liquids as breeder and coolant

Moving electrically conducting fluid ←→ magnetic field

Liquid metal magneto-hydrodynamics (MHD)

Need of electric potential formulation

- ◈ *mhdFoam* available solver

Induction equation
(transport eq. for $\mathbf{B}$)

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = \frac{1}{Re_m}\nabla^2\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v}$$

Magnetic Reynolds number

$$Re_m = \frac{u_0 L}{\boxed{1/\mu\sigma}}$$

Magnetic diffusivity

mhdFoam available solver 😐 → *mhdEpotFoam* **new** solver ☺

Need of electric potential formulation

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Boundary conditions

Fully developed MHD duct flow (induced magnetic field constant along duct axis)

$\Rightarrow$ Local Dirichlet $(B(\Gamma) = const = 0)$ or Neumann $(\partial B/\partial n = 0 \text{ at } \Gamma)$ BCs can be set

$\Rightarrow$ Induced magnetic field serves as streamfunction for current in duct cross-section

3D MHD flow

$\Rightarrow$ No local BCs can be defined → induced field in external space has to be considered ($\mathbf{j}_{ext}$= 0 → Ampère‘s law $\nabla \times \mathbf{B} = 0 \;\rightarrow\; \mathbf{B} = \nabla\psi \;\rightarrow$ define ψ at Γ, outside $\nabla^2\psi = 0$)

Need of electric potential formulation

◇ *mhdFoam* available solver → *mhdEpotFoam* **new** solver

Induction equation
(transport eq. for **B**)

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = \frac{1}{Re_m} \nabla^2 \mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v}$$

Magnetic Reynolds number $Re_m = \dfrac{u_0 L}{1/\mu\sigma}$ ← Magnetic diffusivity

MHD channel flow ⇒ externally applied B

Initial boundary value problem: $\mathbf{B} = f(\mathbf{v})$ is determined depending on the flow field

for $Re_m \ll 1$ (liquid metals, industrial applications) ⇒ $\mathbf{B} \neq f(\mathbf{v}, t)$

$\mathbf{B} \neq f(t)$ → $\partial_t \mathbf{B} = 0$ $\xrightarrow{\nabla \times \mathbf{E} = 0}$ $\mathbf{E} = -\nabla\phi$

→ **Inductionless Approximation** (the magnetic field is not affected by the flow. The induced magnetic field can be neglected.)

Magnetohydrodynamic equations ($Re_m << 1$)

Conservation of

– Momentum

$$\frac{1}{N}\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \frac{1}{Ha^2}\nabla^2\mathbf{v} + \underline{\mathbf{j}\times\mathbf{B}}$$

Lorentz force

– Mass & Charge

$$\nabla\cdot\mathbf{v} = 0,\quad \nabla\cdot\mathbf{j} = 0$$

Ohm‘s law

$$\mathbf{j} = -\nabla\phi + \underline{\mathbf{v}\times\mathbf{B}}$$

Induced electric field

$$\nabla^2\phi = \nabla\cdot(\mathbf{v}\times\mathbf{B})$$

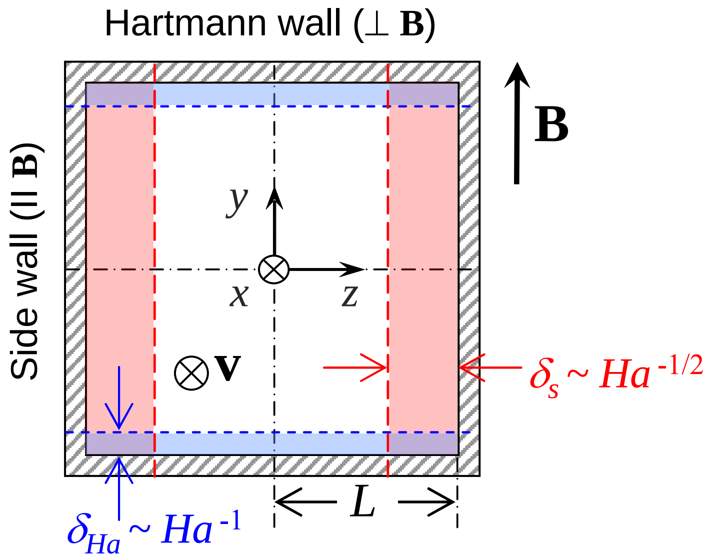
Poisson eq. for ϕ

Dimensionless groups

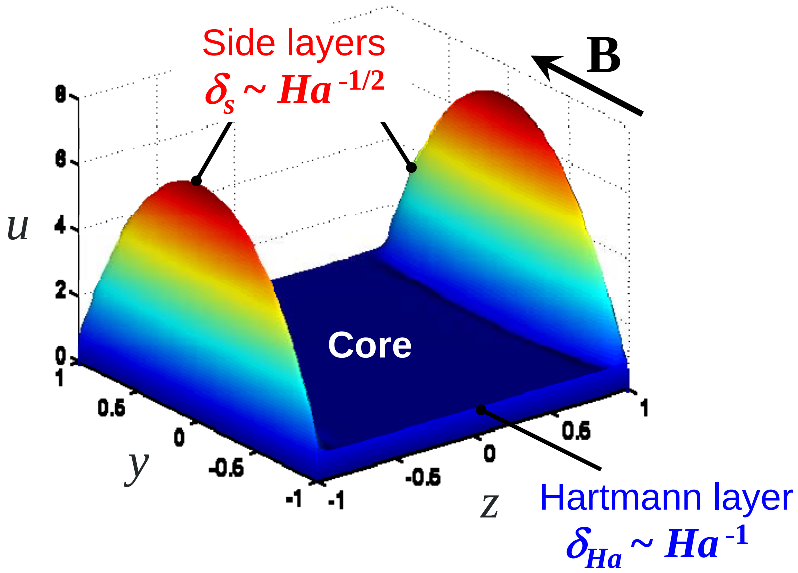
			Fusion reactors
Interaction parameter	$N = \dfrac{\sigma L B^2}{\rho u_0}$	$\dfrac{\text{el.magn. force}}{\text{inertia force}}$	$N \approx 10^5$
Hartmann number	$Ha^2 = \dfrac{\sigma L^2 B^2}{\rho \nu}$	$\dfrac{\text{el.magn. force}}{\text{viscous force}}$	$Ha \approx 10^4$
Reynolds number	$Re = Ha^2 / N$	$\dfrac{\text{inertia force}}{\text{viscous force}}$	

MHD flow features and numerical issues

MHD fully developed flow



Velocity distribution



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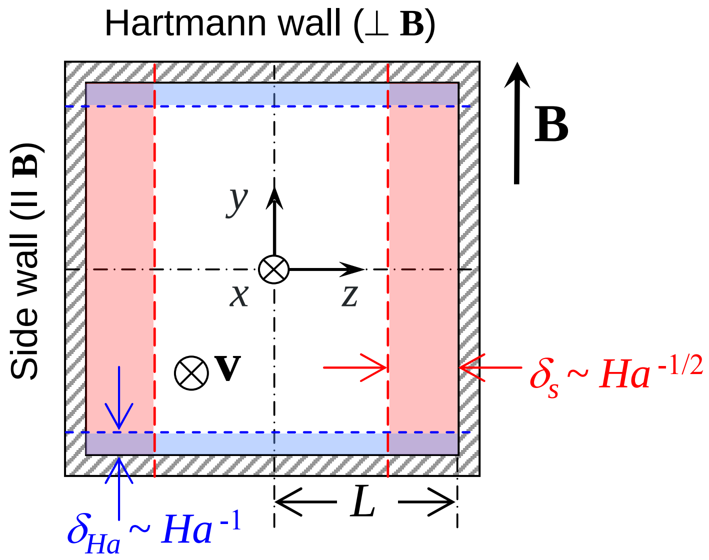

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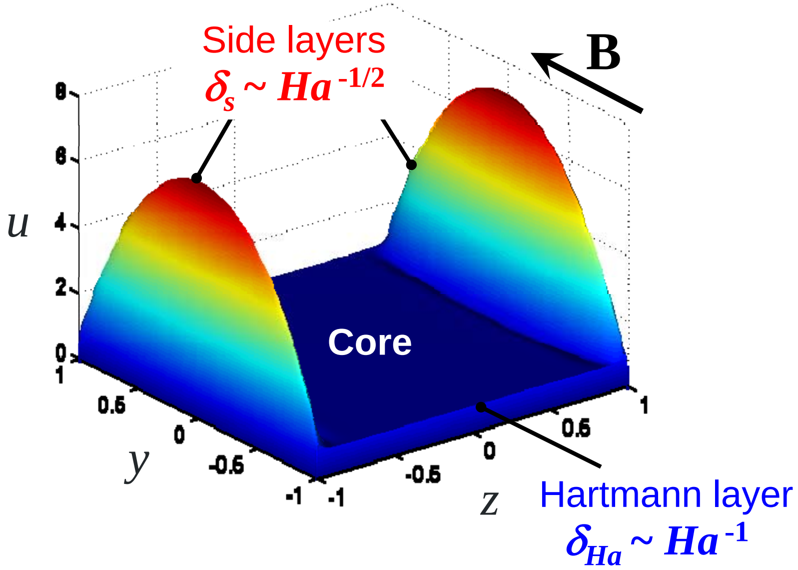


MHD simulation issues: MESH (discretization error)

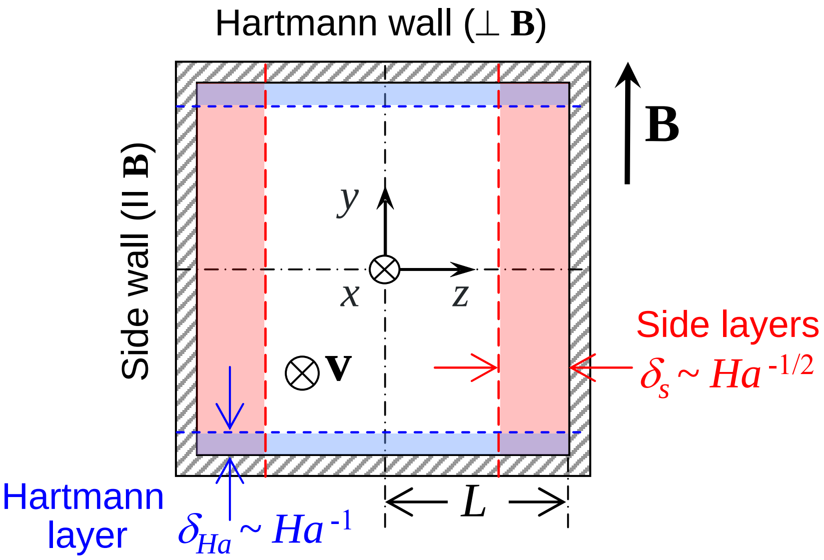
- Suitable resolution of MHD boundary layers
  - Refinement in boundary layers
  - Smooth grid transition between various regions (core - layers)
  - Good cell aspect ratio has to be maintained

  ⇒ By increasing *Ha* (i.e. **B**) the total number of nodes becomes larger
- Walls of finite electric conductivity: strong current turns in the thin wall

  ⇒ The corner region has to be properly resolved

MHD flow features and numerical issues

MHD fully developed flow



Governing equations

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla p + Re^{-1}\nabla^2\mathbf{v} + \boxed{N}\boxed{(\mathbf{j}\times\mathbf{B})}$$

$$\nabla\cdot\mathbf{v} = 0,\ \boxed{\nabla\cdot\mathbf{j} = 0}$$

$$\mathbf{j} = -\nabla\phi + \boxed{\mathbf{v}\times\mathbf{B}}$$

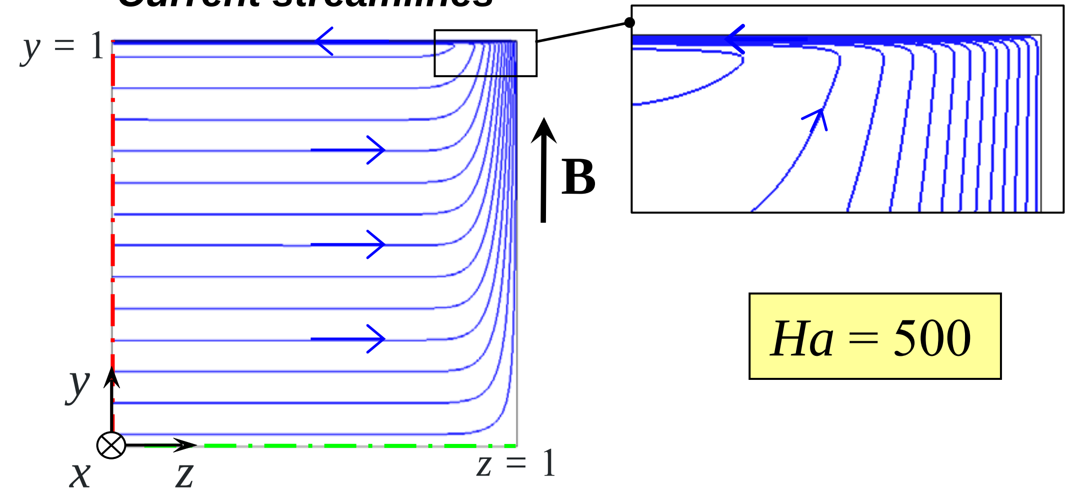
$$\nabla^2\phi = \boxed{\nabla\cdot(\mathbf{v}\times\mathbf{B})}$$

MHD simulation issues: Numerics ←→ Physics (modeling error)

- Error in current density $\mathbf{j}$ is amplified by $\boxed{N}$ in mom. Eq. when used to calculate the Lorentz force
  ⇒ High accuracy required to compute the current density
- Charge conservation has to be ensured: in FVM → balance of fluxes through cell faces
- Source term in ϕ Eq. comes from a part of the current density and has to be given at cell faces
- Lorentz force defined at cell center → proper interpolation of $\mathbf{j}$ from cell face to center

MHD duct flow: electrically insulating walls

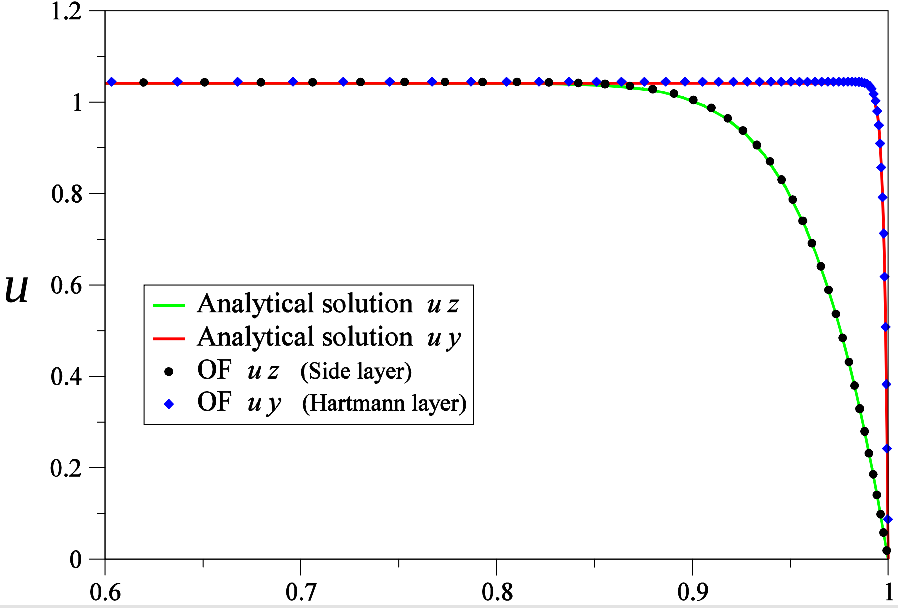
Current streamlines



$Ha = 500$

- Current closes its path in boundary layers (BLs)
- Resolution of BLs critical for the accuracy of the solution (velocity and pressure gradient)

Velocity profile



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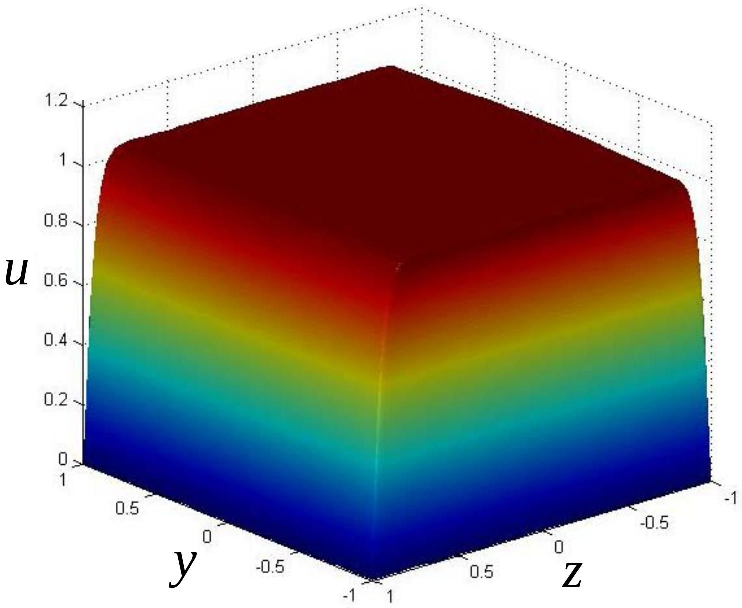


MHD duct flow: walls of finite electric conductivity

conjugateHeatFoam solver (OF 1.5 - dev) → *conjugatemhdFoam* **new** solver

- Momentum Eq. solved only on fluid mesh and ϕ Eq. on both meshes (combined matrix for fluid-solid → *coupledFvScalarMatrix*)

Fluid

$$\mathbf{j} = -\sigma\nabla\phi + \sigma(\mathbf{v}\times\mathbf{B})$$

$$\nabla\cdot(\sigma\nabla\phi) = \nabla\cdot[\sigma(\mathbf{v}\times\mathbf{B})]$$

$$\mathbf{j}\cdot\mathbf{n} = \mathbf{j}_w\cdot\mathbf{n}$$

$$\phi = \phi_w$$

$$\longleftrightarrow$$

Solid / Wall

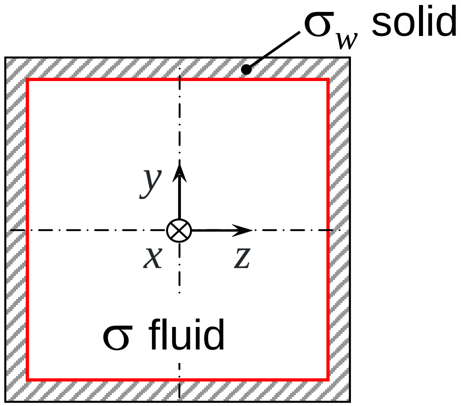
$$\mathbf{j}_w = -\sigma_w\nabla\phi_w$$

$$\nabla\cdot(\sigma_w\nabla\phi_w) = 0$$

σ, σ_w electric conductivity of fluid and wall

- Definition of the electric conductivity <u>field</u> σ
- <u>Coupled boundary conditions</u> for σ and ϕ

```
InterF
{
    type    regionCouple;
    ⋮
}
```



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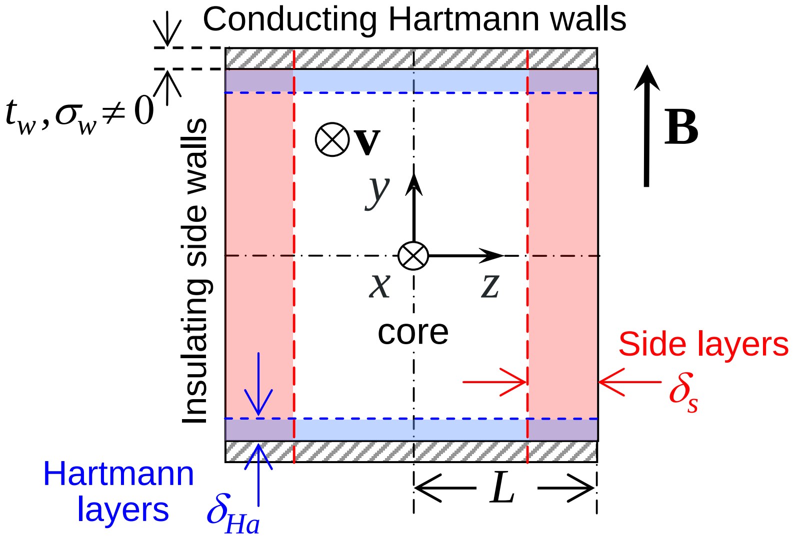
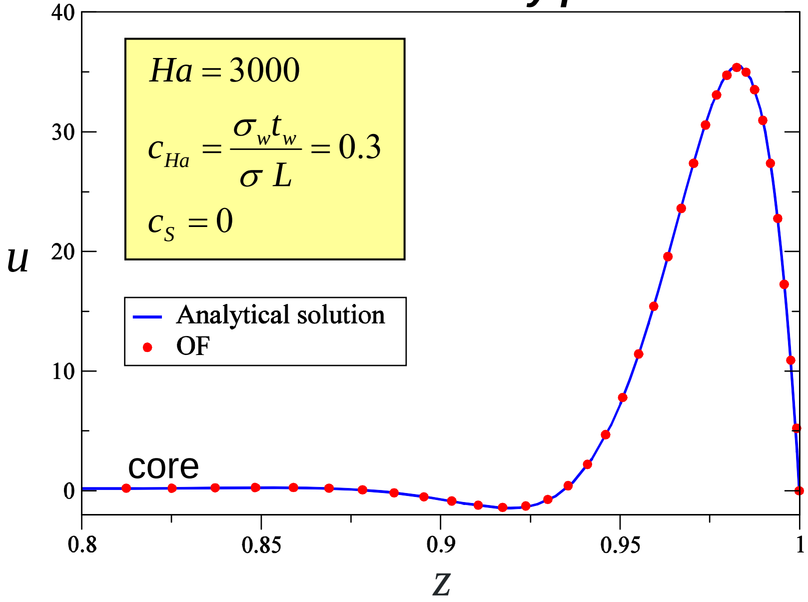
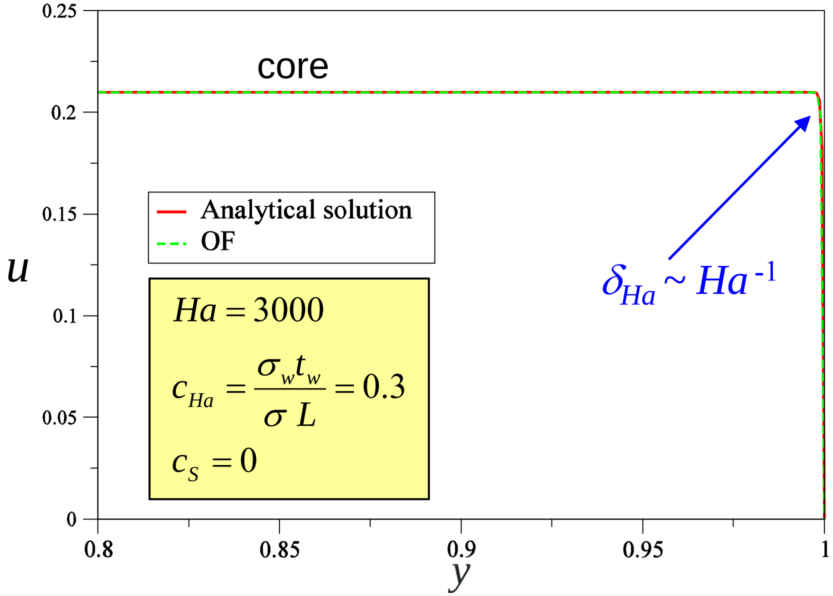


Transverse velocity profile

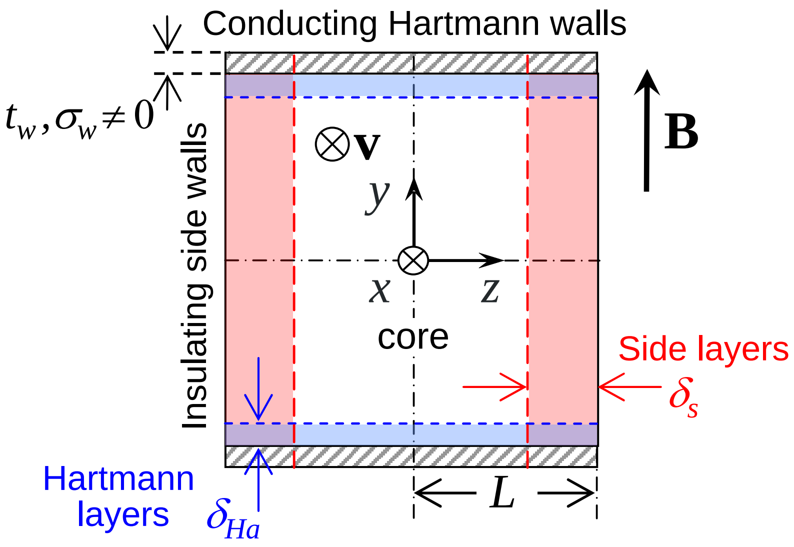


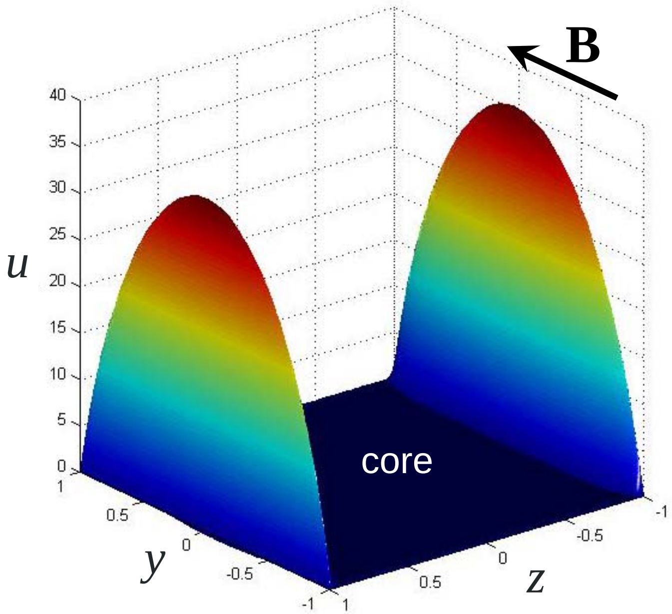
B-aligned velocity profile



MHD duct flow: walls of finite electric conductivity

conjugateHeatFoam solver (OF 1.5 - dev) → *conjugatemhdFoam* **new** solver





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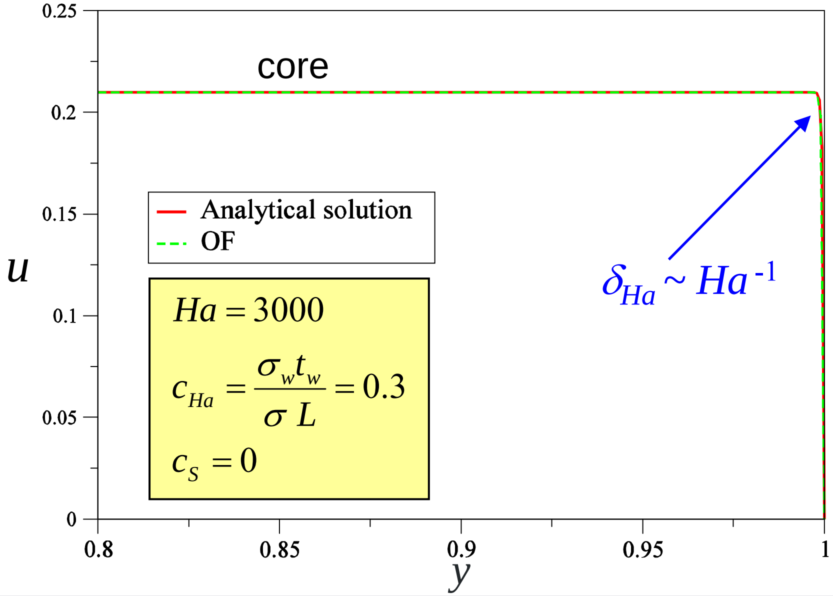
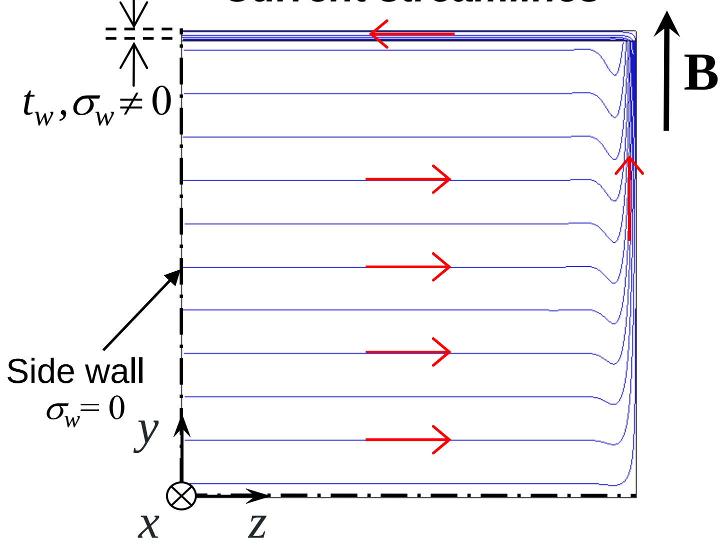


Current streamlines



Summary

- Explanation of need of electric potential formulation to simulate 3D MHD flows
  → difficult BCs for 3D MHD flows in case of induction equation approach
- Description of magneto-hydrodynamic (MHD) equations
  → Lorentz force in mom. Eq., Ohm‘s law, ϕ Poisson Eq.
- Issues for MHD flow simulations:
  - MESH: proper resolution of thin boundary layers, $\delta_{BL} \sim Ha^{-n}$
  - ALGORITHM-MODELING: accuracy of $\mathbf{j}$ prediction, interpolation of $\mathbf{j}$ from cell face to center, charge conservation

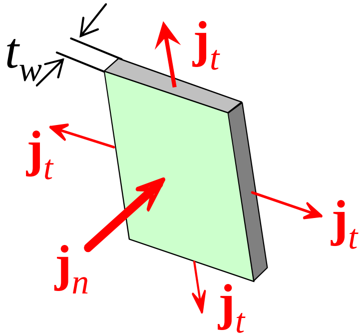
  → new solver *mhdEpotFoam*
- Code validation: perfect agreement with analytical solutions up to $Ha = 5000$
- Successful application to 3D MHD problems
- Channels can have walls of arbitrary electric conductivity
  → new solver *conjugatemhdFoam* from *conjugateHeatFoam* (OF 1.5-dev)

Outlook

- Optimization of present solver version (speed, numerical scheme, grid sensitivity studies, mesh skewness…)
- Development of wall functions (boundary layer models)
- Implementation of thin wall condition

$$\mathbf{j}\cdot\mathbf{n} = -\frac{\partial\phi}{\partial n} = \nabla\cdot\left(c\nabla_t\phi_w\right)$$

Wall element with $c = \dfrac{t_w\sigma_w}{L\sigma}$



- Simulation of MHD flow in ducts with walls of finite electric conductivity
  - 1st approach based on *conjugateHeatFoam* solver (OF 1.5-dev)
    → *conjugatemhdFoam* solver ✓
  - 2nd approach as in *chtMultiRegionFoam* solver (OF 1.6, 1.6.x)
    → *mhdMultiRegionFoam* solver

Cooperation

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